MATH 3 Test 3 – Unit 3
SAMPLE
100 points NAME:______________________
Show all work on this paper. No credit will be given for solutions if work is not shown (except on the first ten problems where it is not necessary to show work). No graphing calculators.

CIRCLE T FOR TRUE, F FOR FALSE. *(2 points each)*

T F (1) $\ln\left(e^x\right) = x$

T F (2) Expanding, $\ln\left(\frac{\sqrt{x}}{y^3 z}\right) = \frac{1}{2}\ln x - 3\ln y + \ln z$

T F (3) $\log_{1/5}\left(25\right) = -2$

T F (4) $\log(x+y) = \log x + \log y$

T F (5) The domain of $f(x) = \log_7(x+4)$ is $(0,\infty)$ $x+4>0$ $x>-4$

Fill in the blanks with the most appropriate answer. *(2 points each)*

(6) The domain of $f(x) = 3 - 4^x$ =is $(-\infty, \infty)$

(7) Without solving for the constants in the numerator, show the initial breakdown of $\frac{3x^2+4x-5}{x^2(x+2)(x^2+3)}$

into partial fractions $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2} + \frac{Dx+E}{x^2+3}$

(8) Show how you would apply the change of base formula to write the expression in terms of natural logs, then use your calculator to approximate it to 3 decimal places,

$\log_4(20)$ in terms of natural logarithms is. $\frac{\ln 20}{\ln 4}$ is approximately = 2.16

(9) $4^{\log_4 64}$ = 64
.

(10) The range of $f(x) = \left(\frac{1}{2}\right)^x$ is $(0, \infty)$.

(11) Combine into a single logarithm: $\frac{1}{3}\log_7(x) - 5\log_7(y)$ *(3 points)*

$\boxed{\log_7\left(\frac{\sqrt[3]{x}}{y^5}\right)}$

(12) Solve each of the following equations : *(9 points)*

(a) $\log_b 125 = 3$ (b) $\log_3 \frac{1}{81} = y$ (c) $\log_{16} x = \frac{3}{2}$

$b^3 = 125$

$\boxed{b = 5}$

$3^y = \frac{1}{81}$

$\boxed{y = -4}$

$16^{3/2} = x$

$x = (\sqrt{16})^3 = 4^3$

$\boxed{x = 64}$

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(13). Solve the following systems of equations. Work should be shown in an organized way so I can follow your thought process. Answers should be given as ordered pairs (5 points each)

(a) $\begin{cases} 3x - 5y = 2 \\ -6x + 10y = 4 \end{cases}$

(Eq 1)(2) $\begin{cases} 6x - 10y = 4 \\ -6x + 10y = 4 \end{cases}$

$0 = 4$

$\boxed{\text{no solution}}$

(b) $\begin{cases} x^2 + y^2 = 16 \\ y = -\frac{1}{6}x^2 \end{cases} \Rightarrow x^2 = -6y$

$-6y + y^2 = 16$

$y^2 - 6y - 16 = 0$

$(y - 8)(y + 2) = 0$

$y = 8 \qquad y = -2$

$x^2 = -48 \qquad x^2 = 12$

no soln. $\qquad x = \pm\sqrt{12} = \pm 2\sqrt{3}$

$\boxed{(\pm 2\sqrt{3}, -2)}$

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(14) Find the partial fraction decomposition of $\dfrac{3x+11}{(x-3)\ (x\ +2)}$ (8 points)

$$\frac{3x+11}{(x-3)\ (x\ +2)} \quad \frac{A}{x-3} + \frac{B}{x+2} = \frac{A(x+2) + B(x-3)}{(x-3)(x+2)}$$

So $3x + 11 = A(x+2) + B(x-3)$

$3x + 11 = (A+B)x + 2A - 3B$

$\begin{cases} 3 = A + B \\ 11 = 2A - 3B \end{cases} \Rightarrow \begin{matrix} B = -1 \\ A = 4 \end{matrix}$

$$\boxed{\frac{4}{x-3} - \frac{1}{x+2}}$$

(15) Solve the following logarithmic equations. *(6points each)*

(a) $\ln(4)-\ln(3x+1)=\ln(2x)$

$\ln \frac{4}{3x+1} = \ln 2x$

$\frac{4}{3x+1} = 2x$

$4 = 6x^2+2x$

$6x^2+2x-4=0$

$3x^2+x-2=0$

$(3x-2)(x+1)=0$

$\boxed{x=2/3}$, $\cancel{-1}$ doesn't check

(b)

$\log_4 2+\log_4(x^2-4)=2$

$\log_4(2x^2-8)=2$

$2x^2-8=4^2$

$2x^2-8=16$

$2x^2-24=0$

$2(x^2-12)=0$

$\boxed{x=\pm\sqrt{12}=\pm 2\sqrt{3}}$

both check

(16) Solve the following exponential equations. (Find the exact solutions, then get an approximation using your calculator) *(4 points each)*

(a) $\left(\frac{1}{2}\right)^{5x+2}=4^{x-3}$

$(2^{-1})^{5x+2}=(2^2)^{x-3}$

$2^{-5x-2}=2^{2x-6}$

$-5x-2=2x-6$

$4=7x$

$\boxed{x=4/7}$

(b) $3^{x+1}=5^{7x}$

$\ln 3^{x+1}=\ln 5^{7x}$

$(x+1)\ln 3=7x\ln 5$

$x\ln 3+\ln 3=7x\ln 5$

$\ln 3=7x\ln 5-x\ln 3$

$\ln 3=x(7\ln 5-\ln 3)$

$\boxed{x=\frac{\ln 3}{7\ln 5-\ln 3}}$

≈ 0.108

(c) $x^2e^{2x}+2xe^{2x}-8e^{2x}=0$

$e^{2x}(x^2+2x-8)=0$

$e^{2x}(x+4)(x-2)=0$

$e^{2x}\neq 0$ $x+4=0$ $x-2=0$

$\boxed{x=-4, x=2}$

(17) Given $f(x) = 3\ln(x+2)$ (11 points)

(a). find $f^{-1}(x)$

$y = 3\ln(x+2)$

$x = 3\ln(y+2)$

$\frac{1}{3}x = \ln(y+2)$

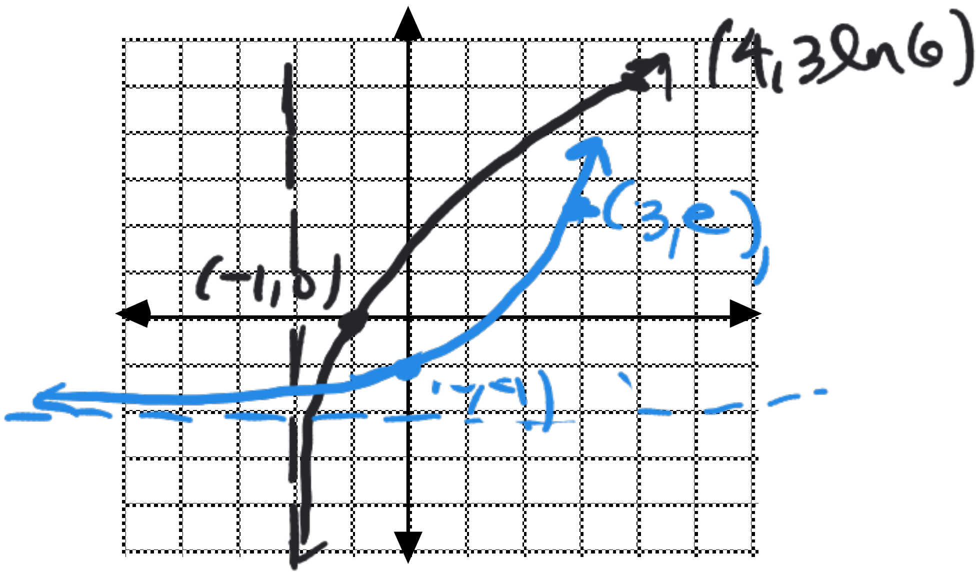
$e^{1/3x} = y+2$

$y = e^{1/3x} - 2$

$$\boxed{f^{-1}(x) = e^{\frac{1}{3}x} - 2}$$

(b) Sketch the graph of $f(x)$ and $f^{-1}(x)$

Show scale. Label two points on graph. Show asymptotes if any.



(c) Find the domain and range of $f(x)$ and $f^{-1}(x)$

	f	f^{-1}
domain	$(-2, \infty)$	$(-\infty, \infty)$
range	$(-\infty, \infty)$	$(-2, \infty)$

(d) Show $f(f^{-1}(x)) = x$

$f(f^{-1}(x)) = f(e^{\frac{1}{3}x} - 2)$

$= 3\ln(e^{1/3x} - 2 + 2)$

$= 3\ln e^{1/3x}$

$= 3(\frac{1}{3}x)$

$= x$

(18). Solve the following system of equations. Work should be shown in an organized way so I can follow your thought process. Answers should be given as ordered triple (8 points)

$$\begin{cases} 2x - 4y + 5z = -33 \\ 4x - y = -5 \\ -2x + 2y - 3z = 19 \end{cases}$$

z alredy eliminated $\rightarrow$ $4x - y = -5$

$\left(-\frac{1}{2}, 3, -4\right)$

eliminate z

(Eq 1)(3) $6x - 12y + 15z = -99$

(Eq 2)(5) $-10x + 10y - 15z = 95$

$\xrightarrow{\text{Add}}$ $-4x - 2y = -4$

$$\begin{cases} 4x - y = -5 \\ -4x - 2y = -4 \end{cases} \xrightarrow{\text{Add}} -3y = -9 \quad y = 3$$

$4x - 3 = -5$

$x = -1/2$

$2x - 4y + 5z = -33$

$-1 - 12 + 5z = -33$

$5z = -20$

$z = -4$

(19) Using Newton's Law of Cooling, answer the following questions (7 points)

$T_0 = 200°F$

$T_s = 70°F$

$D_0 = 200 - 70 = 130°F$

NEWTON'S LAW OF COOLING

If D_0 is the initial temperature difference between an object and its surroundings, and if its surroundings have temperature T_s, then the temperature of the object at time t is modeled by the function

$$T(t) = T_s + D_0 e^{-kt}$$

where k is a positive constant that depends on the type of object.

A cup of coffee has a temperature of 200°F and is placed in a room that has a temperature of 70°F. After 10 min the temperature of the coffee is 150°F. $T(10) = 150$

(a) Find a function that models the temperature of the coffee at time t.

(b) Find the temperature of the coffee after 15 min.

(c) After how long will the coffee have cooled to 100°F?

a) $T(t) = T_s + D_0 e^{-kt} = 70 + 130e^{-kt}$ Use initial condition to find k

$T(10) = 150 \Rightarrow 150 = 70 + 130e^{-k(10)}$

$80 = 130e^{-10k}$

$\frac{8}{13} = e^{-10k} \Rightarrow \ln\left(\frac{8}{13}\right) = -10k \quad k = -\frac{1}{10}\ln\frac{8}{13}$

So $T(t) = 70 + 130e^{-\left(-\frac{1}{10}\ln\frac{8}{13}\right)t}$

$\boxed{T(t) = 70 + 130e^{\frac{1}{10}\ln\left(\frac{8}{13}\right)t}}$

b) $T(15) = 70 + 130e^{\frac{1}{10}\ln\left(\frac{8}{13}\right)15} \approx 132.8°$

c) Find t when $T = 100$

$100 = 70 + 130e$

$\frac{30}{130} = e^{\frac{1}{10}\ln\left(\frac{8}{13}\right)t}$

$\ln\frac{3}{13} = \frac{1}{10}\ln\left(\frac{8}{13}\right)t \qquad t = \frac{10\ln(3/13)}{\ln(8/13)} \approx 30.2 \text{ min}$